The Optical Gravitational Lensing Experiment. Cepheids in the Magellanic Clouds. III. Period-Luminosity-Color and Period-Luminosity Relations of Classical Cepheids*

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ABSTRACT

We present Period-Luminosity-Color and Period-Luminosity relations of classical Cepheids constructed for about 1240 Cepheids from the LMC and 2140 from the SMC. High quality BVI observations (120–360 epochs in the I-band and 15–40 in the BV-bands) were collected during the OGLE-II microlensing experiment. The I-band diagrams of the LMC show very small scatter, $\sigma = 0.074$ mag, indicating that Cepheid variables can potentially be a very good standard candle.

We compare relations of fundamental mode Cepheids from the LMC and SMC and we do not find significant differences of slopes of the Period-Luminosity-Color and Period-Luminosity relations in these galaxies. For the first overtone Cepheids a small change of the slope of Period-Luminosity relation is possible.

We determine the difference of distance moduli between the SMC and LMC with Cepheid relations and compare the result with difference obtained with other standard candles: RR Lyr and red clump stars. Results are very consistent and indicate that the values of zero points of the fundamental mode Cepheid relations are similar in these galaxies. The mean difference of distance moduli between the SMC and LMC is equal to $\mu_{SMC} - \mu_{LMC} = 0.51 \pm 0.03$ mag.

We calibrate the Period-Luminosity-Color and Period-Luminosity relations for classical, fundamental mode Cepheids using the observed LMC relations and adopting the short LMC distance modulus, $\mu_{LMC} = 18.22 \pm 0.05$ mag, resulting from the recent determination with eclipsing system HV2274, RR Lyr and red clump stars.

Finally, we determine a constraint on the absolute magnitude of Cepheids by comparison of their mean V-band magnitude with that of RR Lyr stars in both Magellanic Clouds. The 10-day period, fundamental mode Cepheid is on average 4.63 ± 0.05 mag

*Based on observations obtained with the 1.3 m Warsaw telescope at the Las Campanas Observatory of the Carnegie Institution of Washington.

brighter than RR Lyr stars of LMC metallicity which with the most likely calibration of the brightness of RR Lyr stars yields $M_V^{C,10} = -3.92 \pm 0.09$ mag.

1 Introduction

The Cepheid variable stars are among the most important objects of the modern astrophysics. These relatively well understood pulsating stars provide many empirical information on stellar structure, evolution etc. But what makes them particularly attractive is the well known correlation of their brightness with period, discovered yet at the beginning of the 20th century (Leavitt 1908). This feature of Cepheids and their large absolute brightness make them potentially ideal standard candle for precise distance determination to extragalactic objects where Cepheids have been discovered. However, after almost a century since its discovery the calibration of $P-L$ relation is still a subject of considerable dispute.

While determination of precise periods of Cepheids is straightforward, determination and calibration of Cepheid brightness is difficult. The Galactic Cepheids are located so far from the Sun that distance determination to them can only be obtained by indirect, often very uncertain methods. Even the Hipparcos satellite did not provide much progress in this field as it measured parallaxes of only a few Galactic Cepheids with accuracy better than 30% (Feast and Catchpole 1997). Galactic Cepheids are also usually highly reddened and accurate determination of their brightness is not easy. The Magellanic Clouds, where the $P-L$ relation was discovered, play an important role in this field. Both the Large and Small Magellanic Clouds are known to contain large population of Cepheids, located at approximately the same distance. Therefore the slope of the $P-L$ relation of Cepheids was usually derived based on the Magellanic Cloud objects.

The Magellanic Cloud calibrations (Caldwell and Coulson 1986, Madore and Freedman 1991, Laney and Stobie 1994) are based, however, on limited samples of stars with photometry obtained many years ago mostly with photoelectric and photographic techniques what in crowded fields may lead to systematic uncertainties. Unfortunately both galaxies which are the best objects for studying properties of Cepheids have been very rarely observed with modern CCD techniques. Situation has changed dramatically in 1990s when large microlensing surveys started regular observations of the Magellanic Clouds. Photometry of millions stars is a natural by-product of these surveys, and for the first time precise light curves of thousands of Cepheids could be obtained. The MACHO team (Alcock *et al.* 1995) presented impressive $P-L$ diagram of Cepheids in the LMC showing for the first time clear division between the $P-L$ relation of the fundamental mode and first overtone Cepheids, merged in previous data. Also the EROS group analyzed $P-L$ diagrams in the SMC and LMC (Sasselov *et al.* 1997) suggesting dependence of the zero point of $P-L$ relation on metallicity. They also found a change of slope of the short period fundamental mode Cepheids in the SMC (Bauer *et al.* 1999).

Both the MACHO and EROS data are taken in non-standard bands, therefore they are not suitable for general $P-L$ relation calibrations. The problem of good calibration of the $P-L$ relation becomes very urgent now, because the Cepheid variables are routinely discovered in many galaxies by HST. The main goal of the HST key programme project (Kennicutt, Freedman and Mould 1995) is determination of the Hubble constant. The group uses, however, the universal $P-L$ relation based on very small number of LMC Cepheids (Madore and Freedman 1991), neglecting possible population effects, and assuming the distance modulus to the LMC $\mu_{LMC}=18.50$ mag. Any uncertainties in the calibration of the $P-L$ relation and the LMC distance propagate to any Cepheid based distance and as a result to the value of the Hubble constant with all astrophysical consequences as, for example, age of the Universe.

The Magellanic Clouds were added to the targets of the OGLE microlensing search at the beginning of the second phase of the project – OGLE-II (Udalski, Kubiak and Szymański 1997) in January 1997. Observations are collected in the standard BVI-bands and after more than two years of observations the photometric databases consist of a few hundreds epochs of a few millions stars in the LMC and SMC. The OGLE-II databases were already searched for variable stars and Cepheids are very numerous among them. Some results on Cepheids variables detected in the OGLE-II data – double-mode Cepheids and second overtone Cepheids in the SMC – were already presented in previous papers of this series (Udalski *et al.* 1999a,b). Catalogs with BVI photometry of about 1300 Cepheids from the LMC and 2300 from the SMC will be released in the following papers.

In this paper we present analysis of the period-luminosity-color ($P-L-C$) and $P-L$ relations for Cepheids from the LMC and SMC. We analyze mainly the fundamental mode pulsators because they are used as distance indicators. We compare relations of both the LMC and SMC and find that the slopes of relations in both galaxies are within errors similar. The difference of distance moduli of the SMC and LMC inferred with the Cepheid relations is consistent with that found with other reliable standard candles indicating no dependence of the zero points of Cepheid relations on population differences between the Clouds. We calibrate the $P-L-C$ and $P-L$ relations using the short distance modulus to the LMC ($\mu_{LMC}=18.22$ mag) resulting from the recent determinations with other reliable distance indicators. Finally, we compare V-band magnitudes of Cepheids with that of RR Lyr stars providing a tight constraint on their absolute magnitude.

2 Observational Data

The observational data presented in this paper were collected during the second phase of the OGLE microlensing search with the 1.3-m Warsaw telescope at the Las Campanas Observatory, Chile which is operated by the Carnegie Institution of Washington. The telescope was equipped with the "first generation" camera with a SITe 2048×2048 CCD detector working in the drift-scan mode. The pixel size was 24 μm giving the 0.417 arcsec/pixel scale. Observations were performed in the "slow" reading mode of the CCD detector with the gain 3.8 e^-/ADU and readout noise of about 5.4 e^-. Details of the instrumentation setup can be found in Udalski, Kubiak and Szymański (1997).

Observations covered significant part of the central regions of both the LMC and SMC. Practically the entire bars of these galaxies – more than 4.5 square degrees (21 14.2×57 arcmin driftscan fields) and about 2.4 square degrees (11 fields) were monitored regularly from January 1997 through June 1999 and June 1997 through March 1999 for the LMC and SMC, respectively. Collected BVI data were reduced and calibrated to the standard system. Accuracy of transformation to the standard system was about 0.01 mag. The photometric data of the SMC were used to construct the BVI photometric maps of the SMC (Udalski *et al.* 1998b). The reader is referred to that paper for more details about methods of data reduction, tests on quality of photometric data, astrometry, location of observed fields etc. Quality of the LMC data is similar and it will be fully described with release of the BVI photometric maps of the LMC in the near future.

OGLE-II photometric databases were already searched for variable stars in particular pulsating variables. About 1300 Cepheids were detected in the LMC fields and 2300 in the SMC. Part of the SMC sample, namely double-mode Cepheids and second overtone Cepheids, was already presented in

Udalski *et al.* (1999a, 1999b). The LMC and SMC Cepheid catalogs will be presented in the following papers of this series. They will describe in detail methods of selection of Cepheid variables, determination of mean photometry, completeness of the sample etc.

In short, the light curves of each object consist of about 120–360 epochs in the I-band and about 15-40 in the V and B-bands. BVI photometry was available for all 11 driftscan fields in the SMC. For the LMC the B-band photometry is at the moment of writing this paper less complete – reductions of only 40% of fields were finished. For the remaining fields only VI photometry was available. B-band photometry of these fields will be completed after the next observing season.

The observing data used for construction of the $P-L-C$ and $P-L$ diagrams consist of BVI photometry and period of light variations. The intensity-mean brightness of each object was derived by fitting the light curve with fifth order Fourier series. Accuracy of the I-band mean magnitudes is about a few thousands of magnitude. Accuracy of the BV-band magnitudes is somewhat worse because of worse sampling of light curves – about 0.01 mag. Accuracy of periods is about $7 \cdot 10^{-5} P$.

3 Determination of the $P-L-C$ and $P-L$ Relations

To derive the $P-L-C$ and $P-L$ relations for the Cepheids in the LMC and SMC, we selected all objects from the OGLE Catalogs of Cepheids cataloged as fundamental mode (FU) and first overtone (FO) pulsators. In the next step we corrected the mean brightness for interstellar extinction. We used the red clump stars as the reference brightness for determination of the mean interstellar extinction. The red clump stars are very numerous in both the LMC and SMC, their I-band magnitude was shown to be independent on the age of these stars in the wide range of $2-10$ Gyr, and it is only slightly dependent on metallicity (Udalski 1998a,b). The latter correction is not important in this case because of practically homogeneous environment of field stars of the LMC (Bica *et al.* 1998) or SMC. Thus the mean brightness of the red clump stars can be a very good reference of brightness for monitoring extinction. Similar method was used by Stanek (1996) for determination of extinction map of Baade's Window in the Galactic bulge.

The reddening was determined in 84 lines-of-sight in the LMC and 11 in the SMC. The total mean reddening of the observed fields was found to be

$E(B-V)=0.137$ and $E(B-V)=0.087$ in the LMC and SMC, respectively. Its fluctuations, more significant in the LMC, and non-uniform distribution of Cepheids resulted in somewhat larger total mean reddening of these stars: $E(B-V)=0.147$ and $E(B-V)=0.092$ for the LMC and SMC Cepheid samples, respectively. More details on extinction determination will be provided with the release of Catalogs of Cepheids in the LMC and SMC.

It is obvious that our extinction corrections remove effects of extinction in statistical sense only but because of huge sample of Cepheids presented in this paper our approach is fully justified. As a test whether our extinction correction indeed removes inhomogeneities of extinction we compared standard deviations of the LMC $P-L$ relation constructed for observed magnitudes with those presented below for extinction corrected samples. We found improvement of the standard deviation from $\sigma=0.123$ to $\sigma=0.110$ in the I-band and $\sigma=0.183$ to $\sigma=0.159$ in the V-band for the observed and extinction corrected samples, respectively. This indicates that our extinction procedure works correctly. In the SMC the decrease of standard deviation is smaller due to more uniform extinction there.

It should be, however, stressed here that extinction is variable within each Cloud, growing with the distance inside the Cloud. Therefore in any line-of-sight we may expect an additional scatter of brightness when the mean extinction correction is used. Also the extinction correction we applied was determined from different population of stars than Cepheids, namely old red clump stars. It is possible that the spatial distribution of the red clump stars along the line-of-sight is different than that of much younger Cepheids. Thus, some systematic differences between the determined reddening and the mean reddening of Cepheids in a given direction cannot be ruled out.

After correcting photometry of our samples of Cepheids for interstellar extinction we determined the $P-L-C$ and $P-L$ relation with iterative procedure using the least square method. We fitted the relations in the following form:

$$M = \alpha \cdot \log P + \beta \cdot CI + \gamma \tag{1}$$

for the $P-L-C$ relation, and

$$M = a \cdot \log P + b \tag{2}$$

for the $P-L$ relation. P is the period of Cepheid, M magnitude and CI color index.

After each iteration the points deviating by more than 2.5σ were removed and fitting was repeated. In this way we removed a few outliers – usually the objects reddened significantly more than the mean correction applied to our data or objects blended with background stars. 2.5σ value was selected after a few tests as a compromise value allowing effective removing of outliers and on the other hand not removing too many good objects.

The $P-L-C$ relation was determined for the I-band brightness and $V-I$ color. The $P-L$ relation was constructed for the B, V and I-bands. Additionally we determined the $P-L$ relation for extinction insensitive index W_I (called sometimes Weisenheit index, Madore and Freedman 1991) which is defined as follows:

$$W_I = I - 1.55 * (V - I) \tag{3}$$

The coefficient 1.55 in Eq. 3 corresponds to the coefficient resulting from standard extinction curve dependence of the I-band extinction on $E(V-I)$ reddening (Schlegel, Finkbeiner and Davis 1998). It is easy to show that the values of W_I are the same when derived from observed or extinction free magnitudes, provided that the extinction to the object is not too high so it can be approximated with a linear function of color.

The B-band $P-L$ relation is presented for the SMC only. As we mentioned in Section 2, the LMC sample is less complete in the B-band because only 8 from 21 fields from the LMC have already been reduced in the B-band. Complete sample will be available after the next observing season of the LMC. The B-band sample of the LMC Cepheids presently at our disposal is less than half that numerous than in the VI-bands. In particular the longer period Cepheids, very important for precise determination of the $P-L-C$ and $P-L$ relations, are sparsely populated what would make our determinations less accurate. Therefore to avoid biases we decided to wait with determination of the B-band $P-L$ relation for the LMC until the full data set is available. One has to remember that the B-band $P-L$ relation has intrinsically much larger scatter what makes it much less attractive for distance determination (*cf.* the SMC data). Also, all most important data for extragalactic Cepheids collected by the HST key project team were obtained in the bands closely resembling standard VI-bands, thus precise calibration of $P-L$ relations in these bands is more important.

In this paper we concentrate on analysis of the $P-L$ and $P-L-C$ diagrams of the more important FU mode Cepheids. Cepheids of this type are usually discovered in extra-galactic objects and used for distance determination. We limited our sample of LMC FU mode Cepheids from the short period side at $\log P = 0.4$ for two reasons. First, in the LMC the population of Cepheids with shorter periods is marginal contrary to the SMC where large sample of Cepheids with periods shorter than $\log P = 0.4$ (2.5 days) has been found. Therefore to be able to make non-biased comparison of our relations in both galaxies we limited ourselves to the same range of periods in both galaxies. Secondly, Bauer *et al.* (1999) reported that the slope of the fundamental mode pulsators with periods shorter than 2 days in the SMC is steeper than the longer period stars. Indeed, we also observe in our data such a change of slope. Our lower limit of $\log P = 0.4$ excludes safely this part of the $P-L$ relation of FU mode Cepheids in the SMC. The upper limit of period of our samples is defined by saturation level of the CCD detector because the longest period Cepheids become too bright and are overexposed in our images. It is at $\log P \approx 1.5$ and $\log P \approx 1.7$ for the LMC and SMC, respectively.

4 Discussion

Tables 1 and 3 present results of the least-square (LSQ) fitting of the $P-L$ and $P-L-C$ relations to our samples of classical, fundamental mode Cepheids, respectively. We also list there number of stars used and standard deviation of the residual magnitudes.

We tested stability of our best LSQ solutions performing a few simulations. First, we limited the samples by cutting the lower period limit. Then, we removed randomly significant number (up to the half) of shorter period ($\log P < 0.7$) Cepheids which are much more numerous in both samples than longer period ones. In all cases results were consistent with our best fits for the entire samples differing by not more than ± 0.05 in $\log P$ coefficients of our relations.

We also checked whether our sample of the LMC Cepheids is not severely affected by differential extinction. We performed a series of tests by limiting the sample of Cepheids to those from the fields in which there are indications that the extinction is in the first approximation uniform. The shape of the red clump in the color-magnitude diagram of a given field served as an indicator of how uniform the extinction in the field is. In many fields the shape of the red clump is round indicating little differential extinction. However, in a few cases the oval shape of the red clump, elongated in the direction of reddening, clearly indicates larger differential extinction. We

T a b l e 1

Best least square fit parameters of the $P-L$ relation: $M = a \cdot \log P + b$

LMC – Fundamental Mode Cepheids				
Band	a	b	N	σ
I_0	−2.963	16.560	639	0.110
	0.021	0.015		
V_0	−2.765	17.044	631	0.159
	0.031	0.021		
W_I	−3.278	15.816	669	0.076
	0.014	0.010		
SMC – Fundamental Mode Cepheids				
Band	a	b	N	σ
I_0	−2.857	17.039	488	0.205
	0.033	0.025		
V_0	−2.572	17.480	466	0.257
	0.042	0.032		
B_0	−2.207	17.711	465	0.319
	0.053	0.041		
W_I	−3.303	16.345	469	0.135
	0.022	0.017		

excluded Cepheids located in these fields from our sample. This lowered the number of objects from about 670 to 480. Fitting the $P-L$ and $P-L-C$ relations to such a cleaned sample gave almost identical results as for the full sample for all combination of bands and relations. Thus, our tests indicate that the differential extinction is of little concern in our case.

Simple comparison of results obtained for the LMC and SMC Cepheids allows to draw some conclusions on possible dependence of the $P-L$ relation on differences of metallicity of these galaxies. We find from Table 1 that the slopes, a, of the $P-L$ relation are within errors the same for the I-band and W_I index for the LMC and SMC. Only in the case of the V-band $P-L$ relation we note marginally shallower slope (3.7σ) for the SMC relation as compared to the LMC one. However, taking into account the uncertainty of the true shape of Cepheid extinction which might, for instance, depend slightly on the brightness of Cepheid, the same coefficients for extinction insensitive index W_I and, finally, larger dispersion of the $P-L$ relations in the V-band we do not consider this somewhat lower slope in the SMC as significant. We may conclude that for the metallicity range between the LMC and SMC ([Fe/H]=−0.3, and −0.7 for the LMC and SMC, respectively, Luck *et al.* 1998) the slopes, a, of the $P-L$ relations of fundamental mode classical Cepheids are within errors constant.

Table 2

Final, adopted parameters of the $P-L$ relation:
$M = a \cdot \log P + b$

LMC – Fundamental Mode Cepheids				
Band	a	b	N	σ
I_0	−2.963	16.560	639	0.110
V_0	−2.765	17.044	631	0.159
W_I	−3.278	15.816	669	0.076
SMC – Fundamental Mode Cepheids				
Band	a	b	N	σ
I_0	−2.963	17.115	488	0.207
V_0	−2.765	17.615	466	0.259
W_I	−3.278	16.327	469	0.136

Because the $P-L$ relations of the LMC have much smaller scatter (the standard deviation is almost two times smaller for the LMC relations as compared to the SMC ones) and in the case of the fundamental mode Cepheids of $\log P > 0.4$ they are much better populated, we decided to use the coefficients a derived from the LMC data as universal and we repeated fitting of the SMC data with these coefficients. The fits we obtained are only slightly worse than the best LSQ fits and we treat them as final. Adopted parameters of the $P-L$ relations for the LMC and SMC are listed in Table 2.

We should note at this point that fitting of the $P-L$ relation for the first overtone Cepheids leads to somewhat different results. Although the V and I-band slopes are within errors the same, the more accurate, extinction insensitive index W_I indicates a small difference of slopes of its $P-L$ relation in the LMC and SMC at the 4.5σ level ($a_{W_I}^{LMC} = -3.407 \pm 0.021, a_{W_I}^{SMC} =$

-3.556 ± 0.025). Because the first overtone Cepheids are not used as distance indicators, we only note this small discrepancy, but the problem certainly deserves further studies.

Table 3

Best least square fit parameters of the $P-L-C$ relation: $I_0 = \alpha \cdot \log P + \beta \cdot (V-I)_0 + \gamma$

LMC – Fundamental Mode Cepheids					
Band	α	β	γ	N	σ
I_0	-3.243	1.403	15.887	663	0.074
	0.015	0.026	0.016		
SMC – Fundamental Mode Cepheids					
Band	α	β	γ	N	σ
I_0	-3.487	2.116	16.107	464	0.126
	0.027	0.062	0.032		

Table 3 presents results of the best LSQ fitting of the LMC and SMC $P-L-C$ relations. Comparison of these relations in the LMC and SMC requires special attention. At the first look it may seem that coefficients α and β are different by many sigmas what direct comparison of figures in Table 3 indicates. However, it does not necessarily mean that they are indeed different. It is well known that the α and β coefficients of the $P-L-C$ relation are highly correlated in the sense that the error in α coefficient is coupled with the error of the β coefficient and both errors compensate (Caldwell and Coulson 1986). This makes precise empirical determination of both coefficients difficult but has little consequences on predicted luminosity (and distance determination) if both coefficients come from the same determination. Thus, the α and β should be considered as a pair. To investigate whether the SMC data can be approximated well by the LMC pair of coefficients (α, β), we repeated the $P-L-C$ fitting of the SMC data with α and β fixed from the LMC determination. Results are given in Table 4 and Fig. 1 which presents the $P-L-C$ relation in the form of plot of $I_0 - 1.403 \cdot (V-I)_0$ against $\log P$ for both the LMC and SMC. The fit is somewhat worse than the best LSQ fit of the SMC data ($\sigma = 0.126$ *vs.* 0.138, for the best LSQ and LMC coefficients fit, respectively), but it is clearly seen from Fig. 1, that the pair (α,β) from the LMC fits the SMC data almost equally well. Both sequences of Cepheids for the LMC and SMC in Fig. 1 are within errors parallel indicating that differences between the best LSQ fit and that with LMC coefficients α and β are marginal. Therefore, there is no indication that these coefficients differ significantly between the LMC and SMC.

Table 4

Final, adopted parameters of the $P-L-C$ relation:
$I_0 = \alpha \cdot \log P + \beta \cdot (V-I)_0 + \gamma$

LMC – Fundamental Mode Cepheids					
Band	α	β	γ	N	σ
I_0	−3.243	1.403	15.887	663	0.074
SMC – Fundamental Mode Cepheids					
Band	α	β	γ	N	σ
I_0	−3.243	1.403	16.400	463	0.138

Final, adopted parameters of the $P-L-C$ relation in the LMC and SMC are provided in Table 4. It is worth noting that the color term, β, of the LMC determination (and as a consequence the SMC, because we adopt the LMC values of α and β as universal) is very close to the coefficient of the I-band extinction dependence on $E(V-I)$, 1.55, making the fitting of the $P-L-C$ relation non-sensitive to interstellar extinction.

Figures 2–4 and 5–8 show the $P-L$ relations for LMC and SMC Cepheids, respectively for the BVI-bands and W_I index. In the upper panel of each Figure all observed Cepheids are plotted. Dark and light dots indicate the fundamental mode and first overtone Cepheids, respectively. In the lower panel, the $P-L$ relation for the fundamental mode Cepheids is shown. Dark and light points in the lower panel indicate objects included and rejected from the final fits, respectively. Solid line shows the $P-L$ relation with coefficients adopted from Table 2.

The $P-L-C$ relation (Fig. 1) and W_I index $P-L$ relation (Fig. 4) of the LMC show incredibly small scatter from the fitted relation. The standard

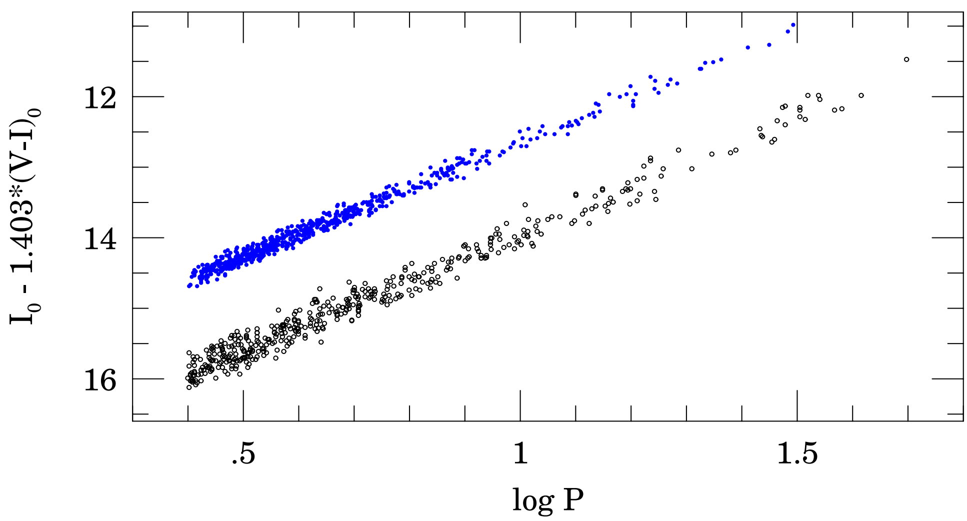

Fig. 1. I-band magnitudes of the fundamental mode Cepheids with subtracted color term resulting from the final $P-L-C$ relation (Table 4). Filled dots and open circles correspond to the LMC and SMC Cepheids, respectively. The SMC diagram is shifted additionally down by 0.8 mag.

deviation of differences between the observed and fitted values is equal to only 0.074 mag and determines most likely the intrinsic dispersion of the $P-L-C$ and $P-L$ relations. It also proves that the Cepheid variable stars can indeed be a very good standard candle allowing precise distance determination good to a few percent. In the case of the SMC the scatter is somewhat larger amounting to $\sigma = 0.126$ mag. This could be expected as it is widely believed that the geometrical depth of the SMC, which is tilted to the line of sight much more than seen almost face-on LMC, is larger than for the LMC (Caldwell and Coulson 1986).

4.1 SMC – LMC Distance Ratio

Comparisons of coefficients a, of the $P-L$ relations and (α, β) of the $P-L-C$ relation in the LMC and SMC indicate no significant change of their values in these galaxies. This is in agreement with most of theoretical modeling (Chiosi, Wood and Capitanio 1993, Saio and Gautschy 1998, Alibert *et al.* 1999) although the opposite predictions can also be found in the literature (Bono *et al.* 1999).

On the other hand, it is believed that metallicity variations among ob-

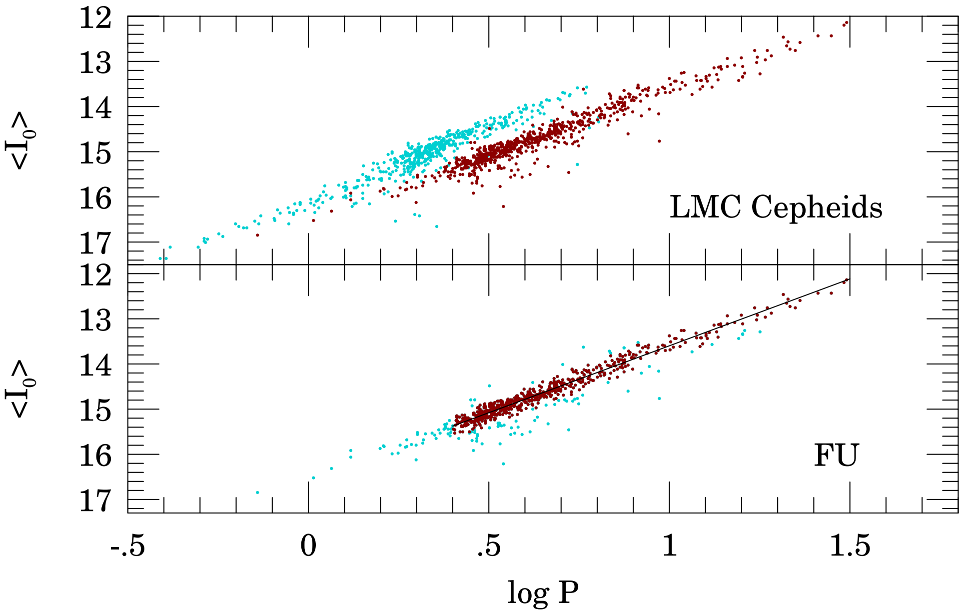

Fig. 2. Upper panel: I-band $P-L$ relation of the LMC Cepheids. Darker and lighter dots indicate FU and FO mode Cepheids, respectively. Lower panel: $P-L$ relation for the FU Cepheids. Solid line indicates adopted approximation (Table 2). Dark and light dots correspond to stars used and rejected in the final fit, respectively.

jects may have stronger effect on the zero point of the $P-L-C$ or $P-L$ relations. Results of previous empirical attempts to determine effects of metallicity on zero points, generally suggested fainter Cepheids in metal poorer objects, however, with high degree of uncertainty (Sasselov *et al.* 1997, Kochanek 1997, Kennicutt *et al.* 1998).

We may test dependence of the zero points of the $P-L-C$ or $P-L$ relations on metallicity in very straightforward manner – by determination of the difference of distance moduli between the Magellanic Clouds resulting from these relations and comparison of results with similar determinations based on other reliable distance indicators observed in both Clouds.

With the $P-L$ relation we may determine the difference of distance moduli of the LMC and SMC for the V and I-bands and, what is more important, for extinction insensitive W_I index which should yield more precise result. Results – the difference of the zero points of corresponding $P-L$ relations in the LMC and SMC (Table 2) are listed in Table 5. We assign lower weight to VI-band determinations because of extinction uncertainty.

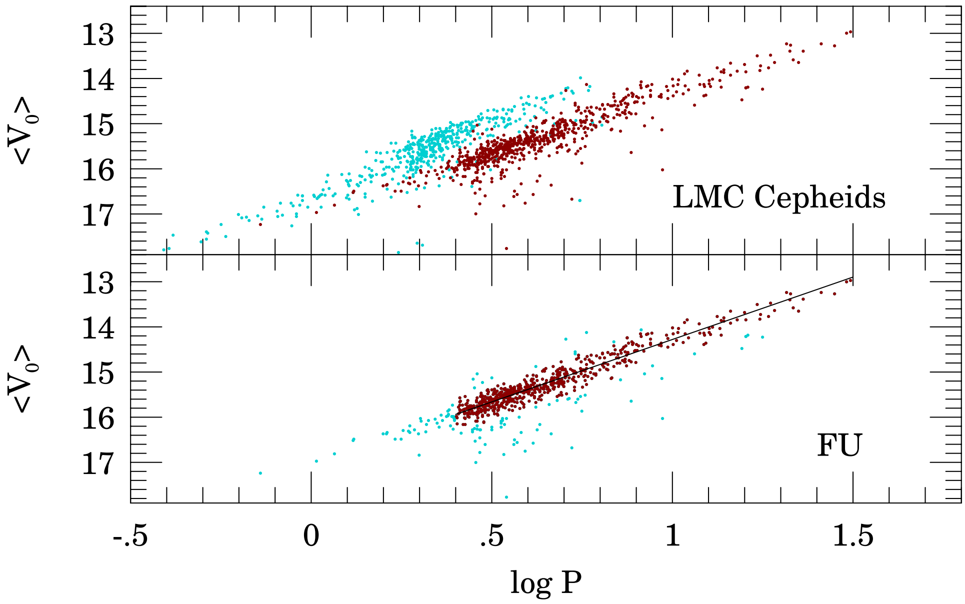

Fig. 3. Upper panel: V-band $P-L$ relation of the LMC Cepheids. Darker and lighter dots indicate FU and FO mode Cepheids, respectively. Lower panel: $P-L$ relation for the FU Cepheids. Solid line indicates adopted approximation (Table 2). Dark and light dots correspond to stars used and rejected in the final fit, respectively.

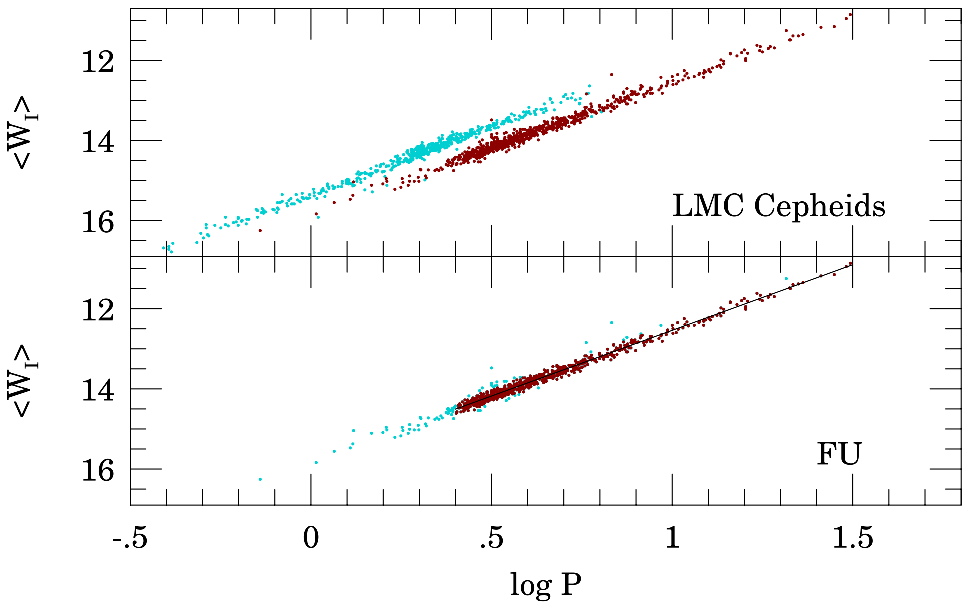

Fig. 4. Upper panel: W_I index $P-L$ relation of the LMC Cepheids. Darker and lighter dots indicate FU and FO mode Cepheids, respectively. Lower panel: $P-L$ relation for the FU Cepheids. Solid line indicates adopted approximation (Table 2). Dark and light dots correspond to stars used and rejected in the final fit, respectively.

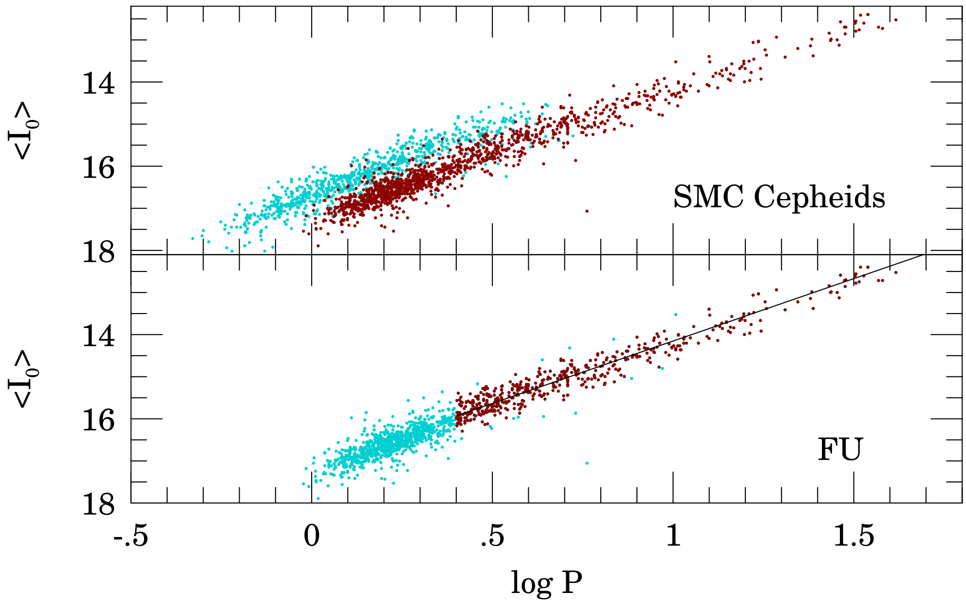

Fig. 5. Upper panel: I-band $P-L$ relation of the SMC Cepheids. Darker and lighter dots indicate FU and FO mode Cepheids, respectively. Lower panel: $P-L$ relation for the FU Cepheids. Solid line indicates adopted approximation (Table 2). Dark and light dots correspond to stars used and rejected in the final fit, respectively.

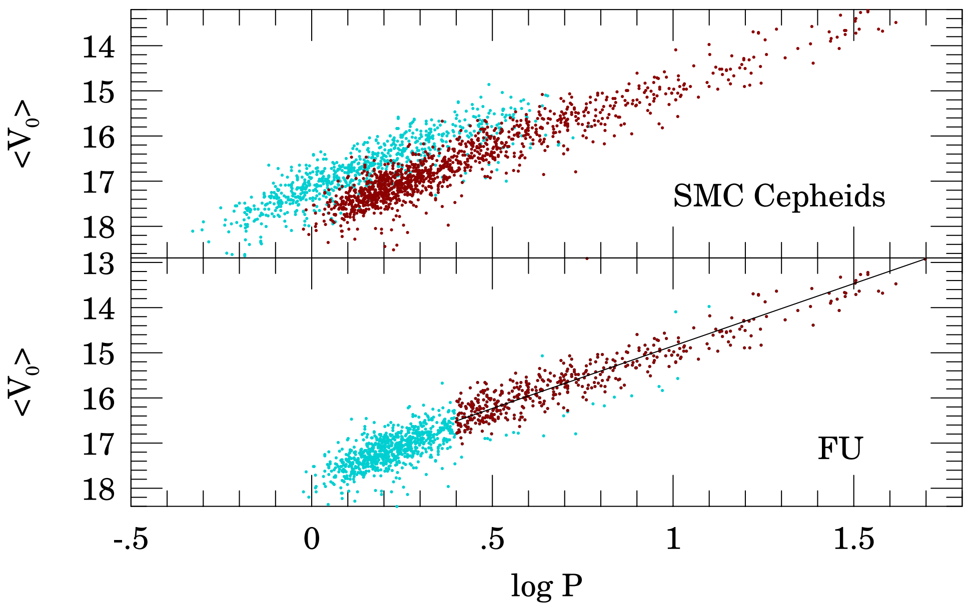

Fig. 6. Upper panel: V-band $P-L$ relation of the SMC Cepheids. Darker and lighter dots indicate FU and FO mode Cepheids, respectively. Lower panel: $P-L$ relation for the FU Cepheids. Solid line indicates adopted approximation (Table 2). Dark and light dots correspond to stars used and rejected in the final fit, respectively.

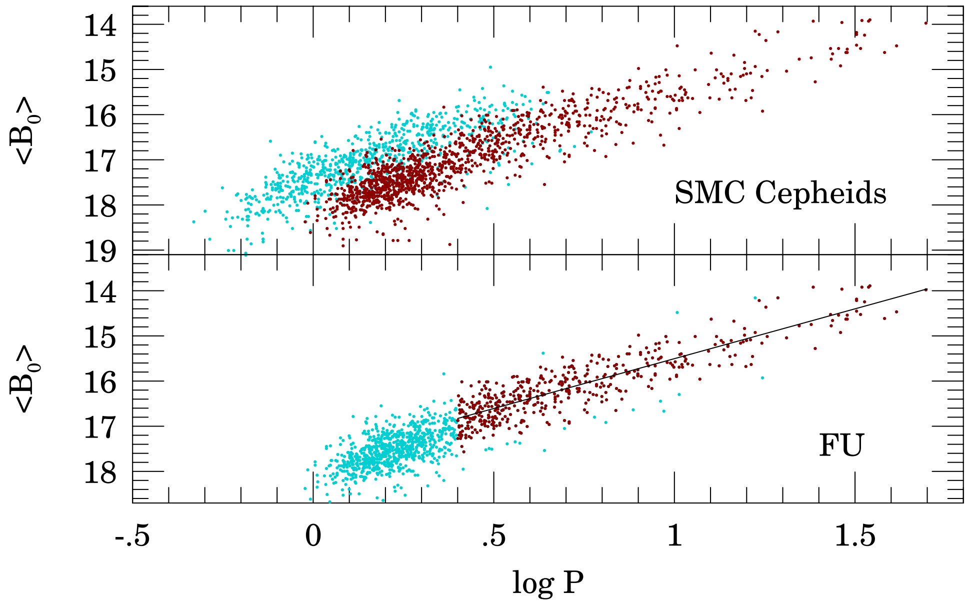

Fig. 7. Upper panel: B-band $P-L$ relation of the SMC Cepheids. Darker and lighter dots indicate FU and FO mode Cepheids, respectively. Lower panel: $P-L$ relation for the FU Cepheids. Solid line indicates adopted approximation (Table 2). Dark and light dots correspond to stars used and rejected in the final fit, respectively.

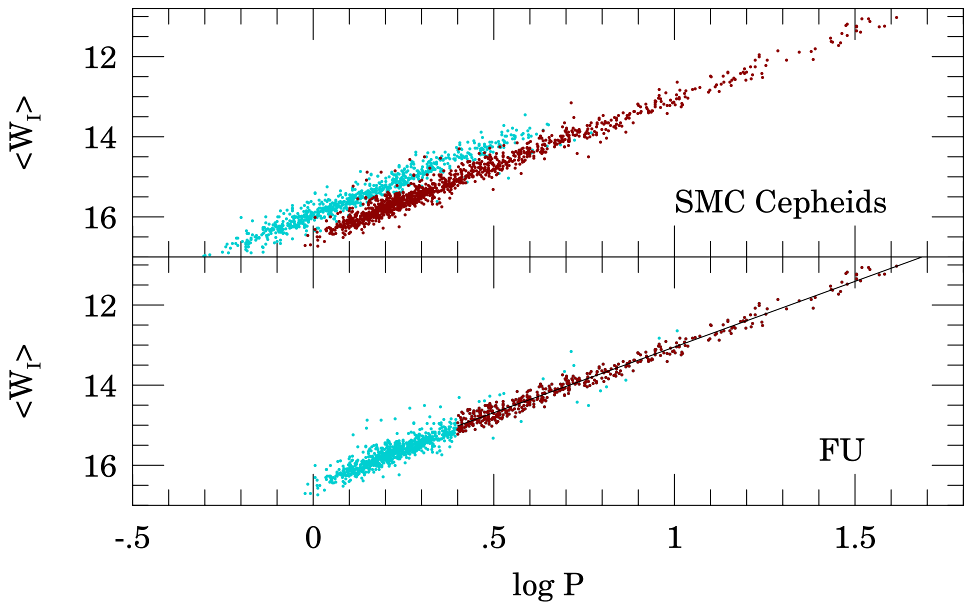

Fig. 8. Upper panel: W_I index $P-L$ relation of the SMC Cepheids. Darker and lighter dots indicate FU and FO mode Cepheids, respectively. Lower panel: $P-L$ relation for the FU Cepheids. Solid line indicates adopted approximation (Table 2). Dark and light dots correspond to stars used and rejected in the final fit, respectively.

The distance determined from the $P-L-C$ relation is also extinction independent. The main source of uncertainty is in this case the color error. We determined the difference of distance moduli for the period corresponding to approximately middle of the relation range: $\log P=1.0$ for the FU Cepheids. The mean $(V-I)_0$ colors of the $\log P=1.0$ fundamental mode Cepheids are $(V-I)_0=0.69\pm0.03$ for the LMC object and $(V-I)_0=0.70\pm0.03$ for the SMC Cepheid. Results of determination with the $P-L-C$ relation are given in Table 5.

Table 5

Difference of distance moduli between the SMC and LMC

Method	Band	$\mu_{SMC}-\mu_{LMC}$	Weight
Cepheid $P-L-C$ (FU)	I	0.53 ± 0.04	1.0
Cepheid $P-L$ (FU)	W_I	0.51 ± 0.03	1.0
Cepheid $P-L$ (FU)	I	0.56 ± 0.06	0.5
Cepheid $P-L$ (FU)	V	0.57 ± 0.08	0.5
RR Lyr stars	V	0.51 ± 0.08	1.0
Red Clump stars	I	0.47 ± 0.09	1.0

To compare results obtained with Cepheids we need independent estimates of difference of the SMC and LMC distance moduli, derived with other reliable standard candles observed in both galaxies. We use values obtained with the red clump and RR Lyr stars. Both types of objects were observed during the OGLE project with the same equipment and methods of reductions. In this way possible systematic errors can be minimized.

Brightness of the red clump stars in the LMC and SMC was derived based on observations of a few star clusters located in the halo of both galaxies where interstellar extinction is small and can be determined from reliable maps of Schlegel *et al.* (1998). The red clump mean I-band, extinction free brightness of the red clump stars is equal to $I_0=17.88\pm0.05$ and $I_0=18.31\pm0.07$ for the LMC and SMC clusters, respectively (Udalski 1998b). With a small correction for the difference of metallicity of clusters (Udalski 1998a), the difference of distance moduli between the LMC and SMC from the red clump stars is $\Delta\mu_{RC}=0.47\pm0.09$ mag.

Preliminary analysis of RR Lyr stars from the OGLE fields was presented in Udalski (1998a). The samples of more than 100 objects in each galaxy are small as compared to the total number of a few thousands found in the entire observed area of the Magellanic Clouds, nevertheless they allow to determine reliable brightness of these objects. Results of the LMC RR Lyr stars presented by Udalski (1998a) were based on moderate number of observations of these stars and preliminary photometric calibrations of the observed fields. Also, for both the LMC and SMC RR Lyr stars, the extinction was estimated by extrapolation or interpolation of available extinction maps. Now, with about three times that many observing epochs available for these LMC RR Lyr stars (about 140 in the I-band and 20 in the V-band) and extinction independently determined we have reanalyzed the objects of Udalski (1998a).

New photometry of 104 RR Lyr from the LMC with appropriate extinction correction yields $\langle V_{RR}^{LMC}\rangle = 18.94 \pm 0.04$ mag. This is practically the same result as presented by Udalski (1998a) ($\langle V_{RR}^{LMC}\rangle = 18.86$ mag), taking into account that extinction was slightly overestimated in that paper. It is in very good agreement with the mean brightness of the RR Lyr stars in a few star clusters in the LMC (Walker 1992). Correction of the mean brightness of RR Lyr stars in the SMC is almost negligible: $\langle V_{RR}^{SMC}\rangle = 19.43 \pm 0.03$ mag as compared to $\langle V_{RR}^{SMC}\rangle = 19.41$ mag in Udalski (1998a).

To derive the difference of distance moduli between both galaxies we have to correct also the brightness of RR Lyr stars for metallicity differences. Fortunately, the mean difference of metallicity of RR Lyr stars in the LMC and SMC is not large (on average: [Fe/H] $= -1.6$ and -1.7 for the LMC and SMC, respectively, see discussion in Udalski 1998a) and with the average slope of the brightness-metallicity relation equal to 0.2 it leads to a small correction of 0.02 mag. The SMC RR Lyr stars would be fainter if they were of the LMC metallicity. Thus, the difference of distance moduli between the SMC and LMC resulting from RR Lyr stars is equal to $\Delta\mu_{RR} = 0.51 \pm 0.08$ mag. It should be noted that the mean brightness of RR Lyr stars in both galaxies will be finally refined when the OGLE catalog of these stars is released.

Results of determinations of distance moduli difference are summarized in Table 5. It can be seen that all determinations based on Cepheids are consistent. The Cepheid determination is in excellent agreement with independent estimates from the red clump and RR Lyr stars. This result indicates that within uncertainty of a few hundredth of magnitude the zero points of the $P-L-C$ and $P-L$ relations are independent of metallicity of hosting object. Thus, the population effects on the Cepheid distance scale are negligible, at least for the metallicity range bracketed by the Magellanic Clouds. The mean difference of the SMC and LMC distance moduli resulting from three independent standard candles is $\mu_{SMC}-\mu_{LMC}=0.51\pm0.03$ mag.

4.2 Calibration of the $P-L-C$ and $P-L$ Relations

The traditional way of calibrating the standard candles is based on observations of the same type objects located in the Galaxy. In the case of Galactic Cepheids such a calibration is, however, very difficult. Even the closest Cepheids are located, unfortunately, so far from the Sun that the distance determination can only be performed with indirect methods. Even Hipparcos did not provide parallaxes precise enough to allow unambiguous distance determination for sound sample of these stars. It seems that much better results might be achieved by observations of Cepheids in other galaxies and calibrating the $P-L$ relation based on other, reliable distance determinations. One of such galaxies might be NGC4258 to which very precise distance was recently determined with geometric method, based on maser observations (Herrnstein *et al.* 1999). The galaxy possesses a population of Cepheids detected with HST (Maoz *et al.* 1999). However, although NGC4258 might be a very attractive object for testing and checking the calibration of Cepheids it is certainly not the best object for deriving precise calibration. The Cepheid sample there is small and it may be biased by many factors including difficulty of detecting short period Cepheids, quality of HST photometry etc.

On the other hand the Magellanic Clouds seem to be the best objects to calibrate the Cepheid $P-L$ relation. They are close enough and contain thousands of Cepheids allowing analysis of a large, homogeneous and photometrically accurate sample of these objects, like the one presented in this paper. They are also chemically homogeneous what minimizes uncertainties resulting from metallicity variations (Luck *et al.* 1998).

Unfortunately, the distance to the LMC has been a subject of dispute for a long time. It seems, however, that recent results obtained with different techniques converge at the short distance modulus of $\mu_{LMC}=18.2-18.3$ mag. The most promising, largely geometric method using eclipsing binary stars should allow to derive the distance to the LMC with accuracy of $1-2$ percent. First determination of the distance to the LMC with HV2274 eclipsing system yields the distance modulus of $\mu_{LMC}\approx18.26$ mag (Guinan *et al.* 1998, Udalski *et al.* 1998c). RR Lyr stars calibrated with the most reliable methods (Popowski and Gould 1998) give the distance modulus of

$\mu_{LMC} = 18.23 \pm 0.08$ mag when these calibrations ($M_V^{RR} = 0.71 \pm 0.07$ mag) are used with the mean extinction free V-band magnitude of RR Lyr stars in the LMC presented in the previous Subsection. Finally, the recent distance determination with the red clump stars used as a standard candle (Udalski *et al.* 1998a, Stanek, Zaritsky and Harris 1998) corrected for small population effects (Udalski 1998a,b) yields the distance modulus of $\mu_{LMC} = 18.18 \pm 0.06$ mag. It should be noted that the red clump stars are at present the most precisely calibrated standard candle because very accurate parallaxes (accuracy better than 10%) were measured for hundreds of them in the solar neighborhood by Hipparcos.

We should also mention here the method that is in principle a precise geometric technique – observations of the light echo from the ring of gas observed around the supernova SN1987A. Unfortunately in the case of SN1987A this technique suffers from insufficient quality of observations at crucial moments after the supernova explosion and many modeling assumptions. Only the upper limit of the distance modulus to the LMC can be estimated. It ranges from $\mu_{LMC} < 18.58$ mag (Panagia 1998) to as low as $\mu_{LMC} < 18.37$ mag (Gould and Uza 1998).

To calibrate the Cepheid $P-L-C$ and $P-L$ relations we adopted the average distance modulus resulting from all these determinations: $\mu_{LMC} = 18.22 \pm 0.05$ mag. Any refinement of this value in the future will correspond to the appropriate shift of the zero points of our calibration. Coefficients of the resulting absolute magnitude $P-L-C$ and $P-L$ relations for classical, FU mode Cepheids are given in Table 6.

It should be noted here that the W_I index seems to be a very attractive alternative for distance determination of extra-galactic distances. Instead of V and I-band comparisons, which require extinction estimate, the W_I index seems to be free from these uncertainties. Due to close coincidence of the extinction coefficient and the color term of the $P-L-C$ relation for the I-band and $(V-I)$ color, the dependence on color is simultaneously removed. As a result the $P-L$ relation for this index is much tighter, practically as tight as the $P-L-C$ relation – also making distance determination more precise. One should be, however, aware that extinction is a linear function of the color index for small reddening only. Therefore calculation of the W_I index from observed VI measurements is accurate only for moderately obscured objects. For example, for highly reddened Galactic Cepheids it may still be necessary to calculate W_I index from extinction free V and I magnitudes.

Finally, based on our photometry of Cepheids and RR Lyr stars in both

Table 6

Absolute calibration of the $P-L-C$ and $P-L$ relations with $\mu_{LMC}=18.22$ mag

$P-L-C$ Relation

Fundamental Mode Cepheids			
Band	α	β	γ
M_I	-3.243	1.403	-2.33

$P-L$ Relation

Fundamental Mode Cepheids		
Band	a	b
M_I	-2.963	-1.66
M_V	-2.765	-1.18
W_{M_I}	-3.278	-2.40

Magellanic Clouds we may provide a constraint on the absolute magnitude of fundamental mode Cepheids. The mean V-band magnitude of the 10-day period Cepheid is equal to $\langle V_{C,10}^{LMC}\rangle=14.28\pm0.03$ mag in the LMC and $\langle V_{C,10}^{SMC}\rangle=14.85\pm0.04$ mag in the SMC. The extinction free brightness of other standard candle – RR Lyr stars can be used for comparison. In the previous Subsection we provided appropriate brightness of RR Lyr stars in both galaxies: $\langle V_{RR}^{LMC}\rangle=18.94\pm0.04$ mag and $\langle V_{RR}^{SMC}\rangle=19.43\pm0.03$ mag for the LMC and SMC, respectively. Including a small correction of RR Lyr brightness due to metallicity differences, we find $\Delta V_{RR-C,10}$=4.66±0.05 mag and $\Delta V_{RR-C,10}=4.60\pm0.05$ mag for the LMC and SMC, respectively. The difference is in respect to the RR Lyr star of the LMC metallicity ([Fe/H]$=-1.6$ dex).

Consistent results in both Magellanic Clouds indicate that both Cepheids and RR Lyr are good standard candles. Assuming the absolute calibration of RR Lyr stars: M_V^{RR}=0.71±0.07 mag (Popowski and Gould 1998) we obtain $M_V^{C,10}=-3.92\pm0.09$ for 10-day period Cepheid. Such an absolute brightness of Cepheids and our observed $P-L$ relation ($\langle V^{LMC}_{C,10}\rangle = 14.28$ mag) indicate the distance modulus to the LMC $(m-M) = 18.20$ mag fully consistent with the short distance modulus adopted for absolute calibration *via* the LMC (Table 6). Thus, the distance scale of Cepheids is consistent with distance scales inferred from RR Lyr stars and other reliable distance indicators.

We may also compare the Cepheid absolute magnitude resulting from the RR Lyr calibration with results of studies of Galactic Cepheids. The Galactic calibrations of Cepheids fall into three categories: based on Hipparcos direct parallaxes (Feast and Catchpole 1997, Lanoix, Paturel and Garnier 1999), classical, pre-Hipparcos ones (Laney and Stobie 1994, Gieren *et al.* 1998) and based on statistical parallaxes (Luri *et al.* 1998). They give the brightest, moderate and faintest luminosity of Cepheids at a given period and as a consequence the long, classical and short distance to the LMC. We will not discuss in detail any of these calibrations here.

As we already mentioned the Hipparcos parallaxes of Cepheids are very uncertain and may be biased by many factors. Analysis of the Hipparcos data by Feast and Catchpole (1997) and Lanoix *et al.* (1999) lead to essentially the same results that the absolute V-band magnitude of the Galactic Cepheids with 10-day period is about -4.22 mag. The classical calibration predicts the mean absolute magnitude of the Galactic Cepheids of 10-day period equal to $M_V^{C,10} = -4.07$ mag (Laney and Stobie 1994, Gieren *et al.* 1998). Finally, the statistical parallaxes method predicts much fainter Cepheids: $M_V^{C,10} = -3.86$ mag for 10-day period object (Luri *et al.* 1998).

Comparing these calibrations with the absolute magnitude inferred from comparison of Cepheids with RR Lyr stars in the Magellanic Clouds and the most likely calibration of RR Lyr stars we find that the Galactic calibration based on statistical parallaxes (Luri *et al.* 1998) is closest to our result. It is worth noting that statistical parallaxes method for both Cepheids and RR Lyr stars give consistent results. Calibration of RR Lyr of Popowski and Gould (1998) is based, among others, on statistical parallaxes determination.

Acknowledgements. We would like to thank Prof. Bohdan Paczyński for many discussions and help at all stages of the OGLE project. We thank Dr. K.Z. Stanek for comments and remarks on the paper. The paper was partly supported by the Polish KBN grant 2P03D00814 to A. Udalski and 2P03D00916 to M. Szymański. Partial support for the OGLE project was provided with the NSF grant AST-9530478 and AST-9820314 to B. Paczyński.

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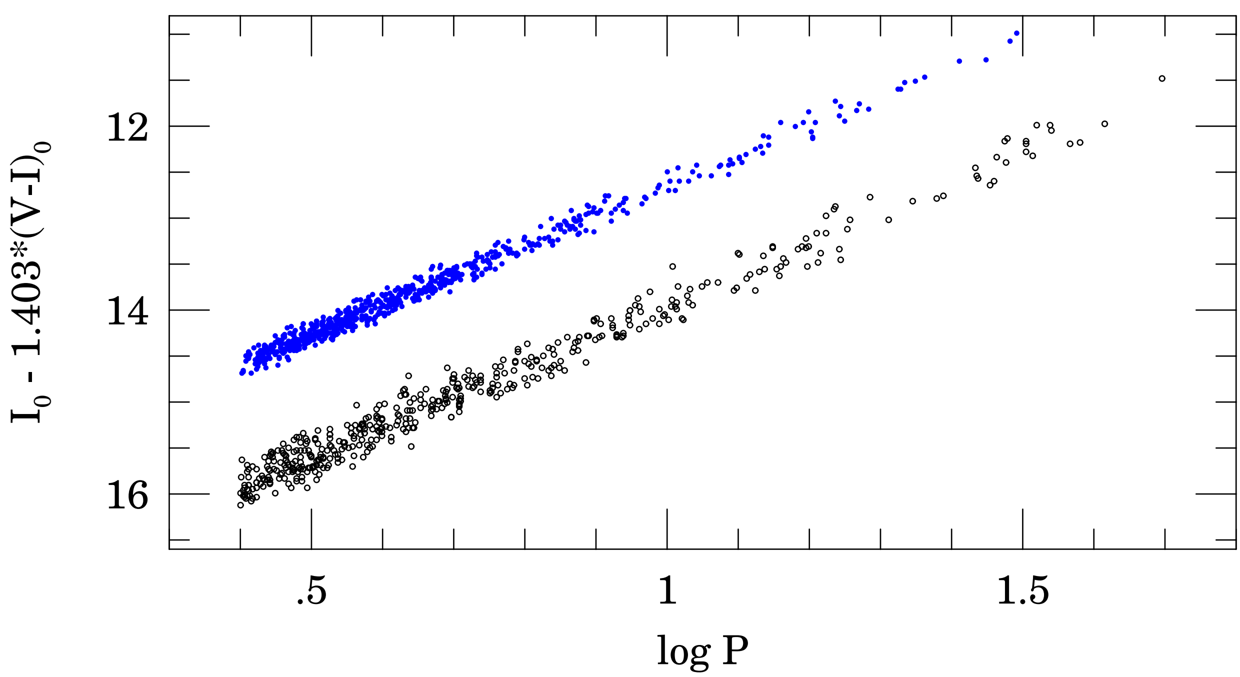

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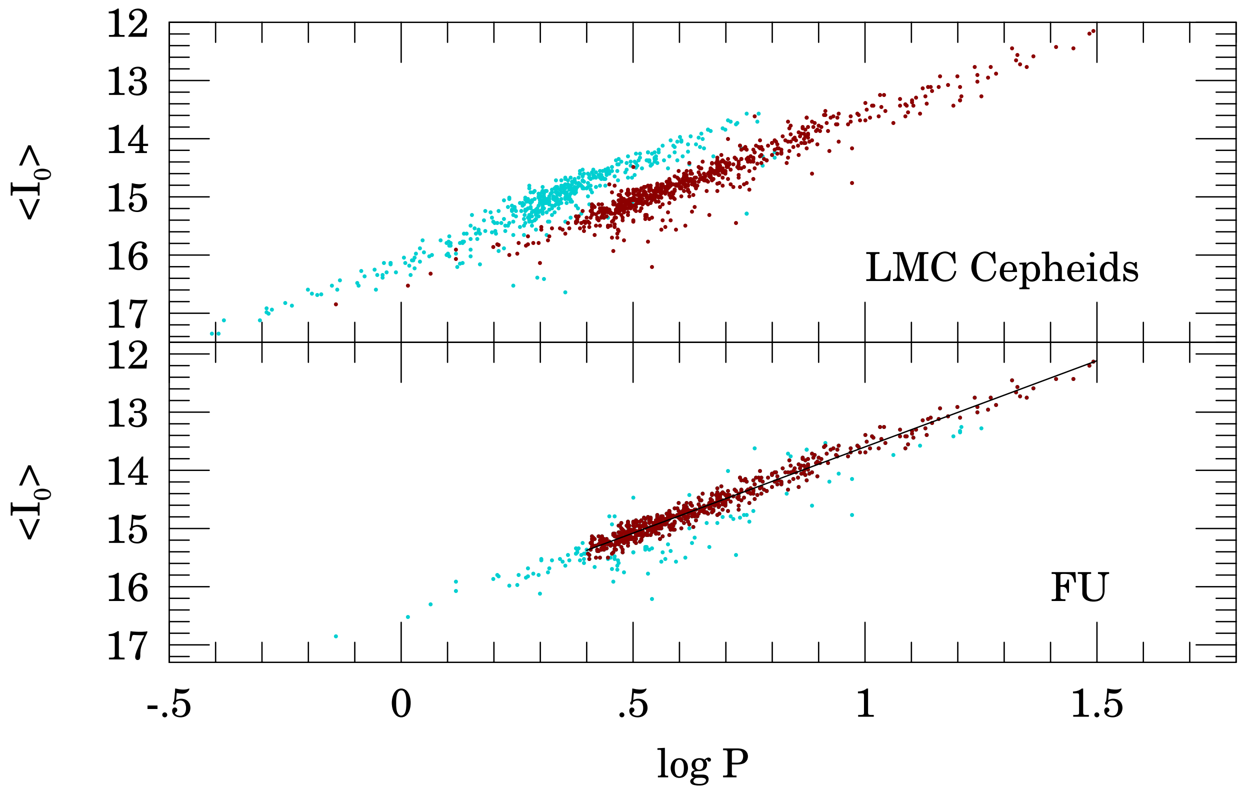
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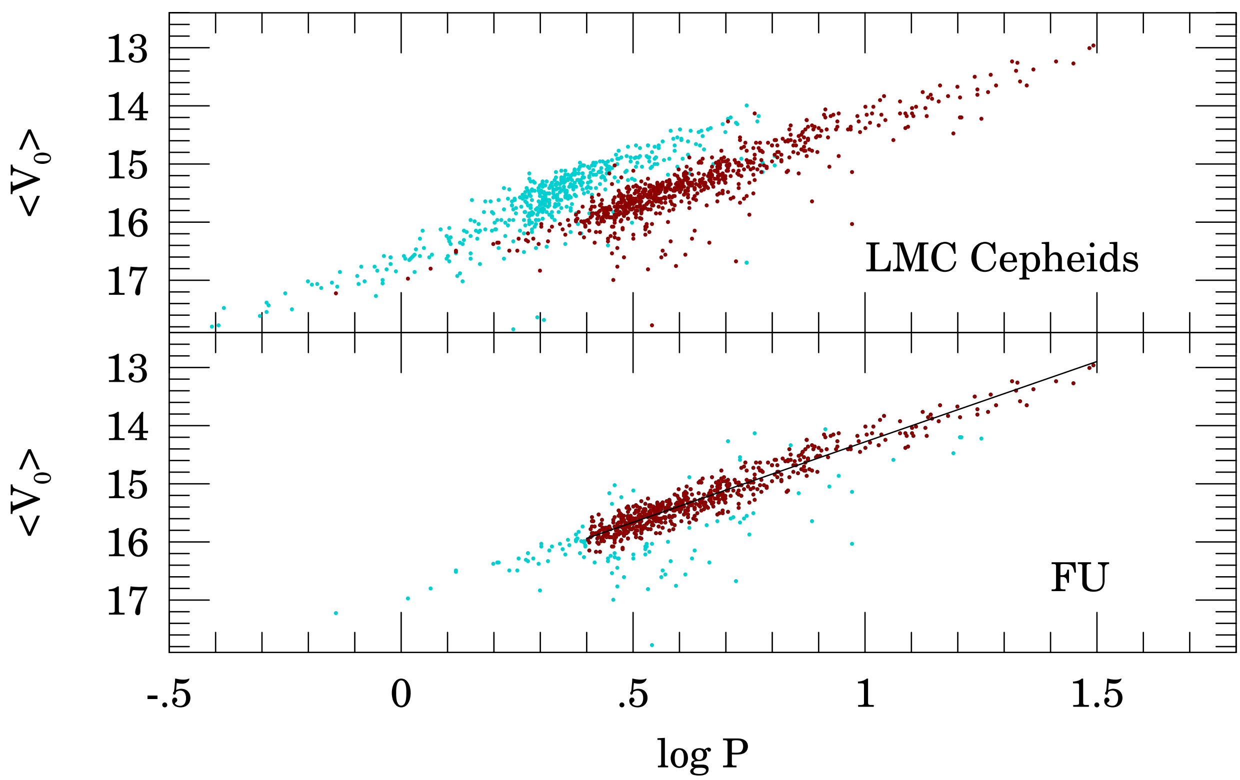

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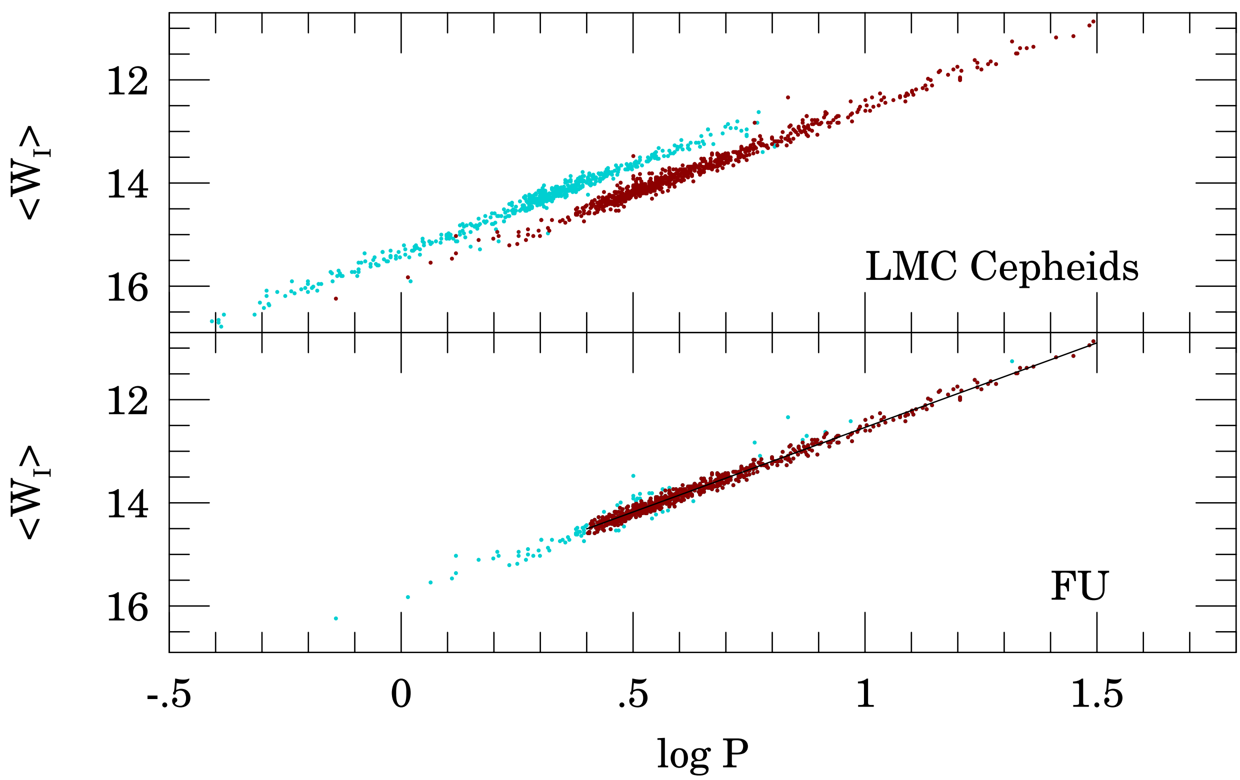

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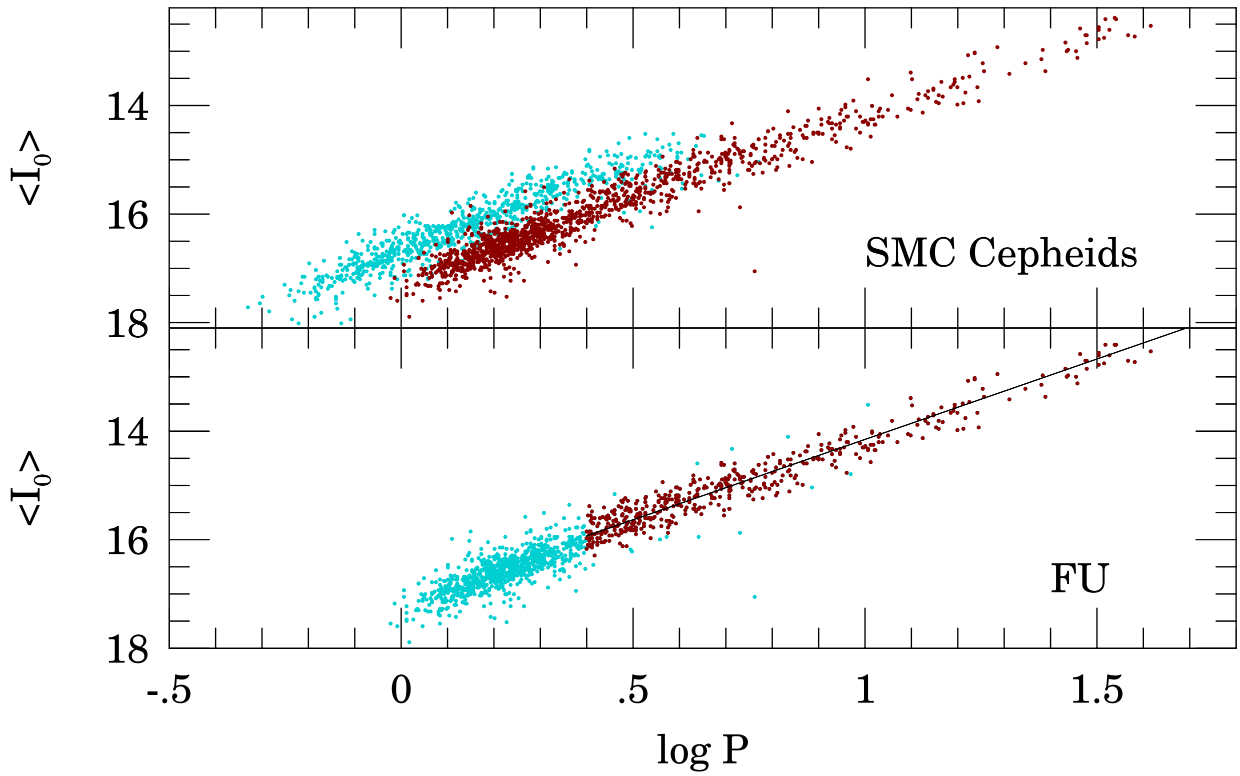

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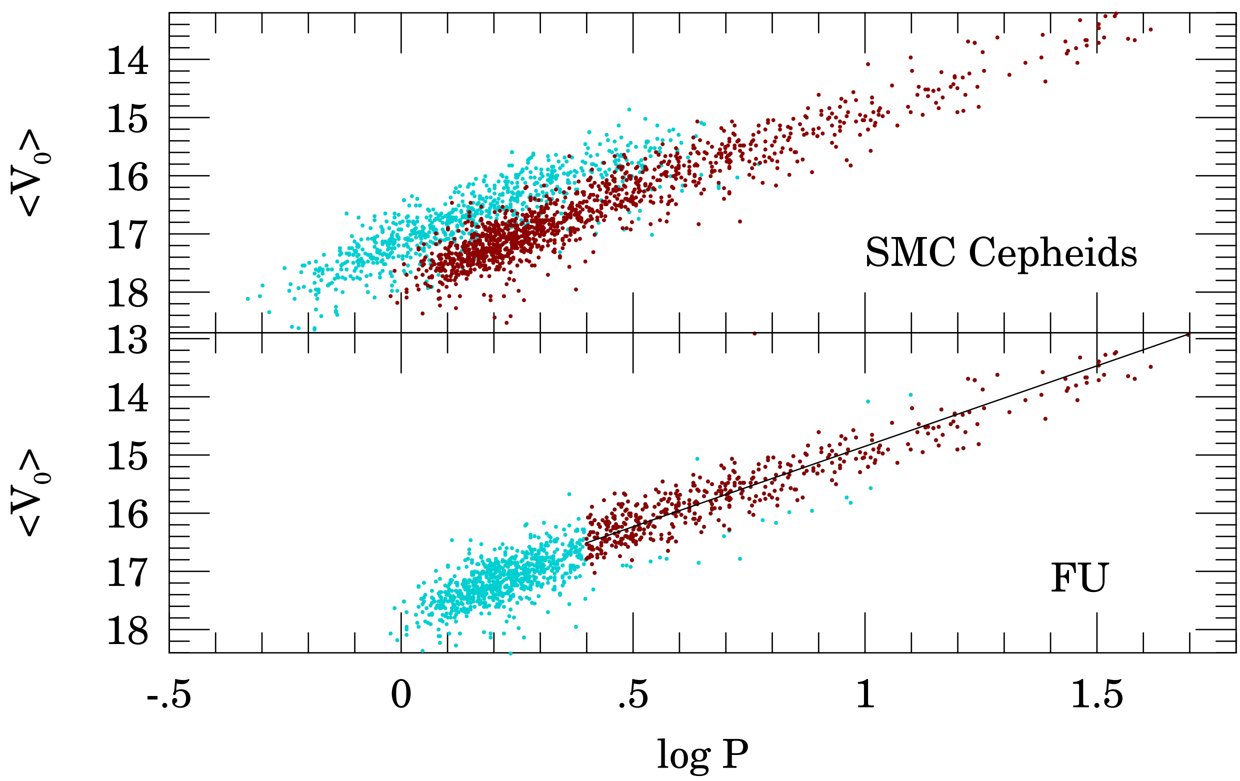

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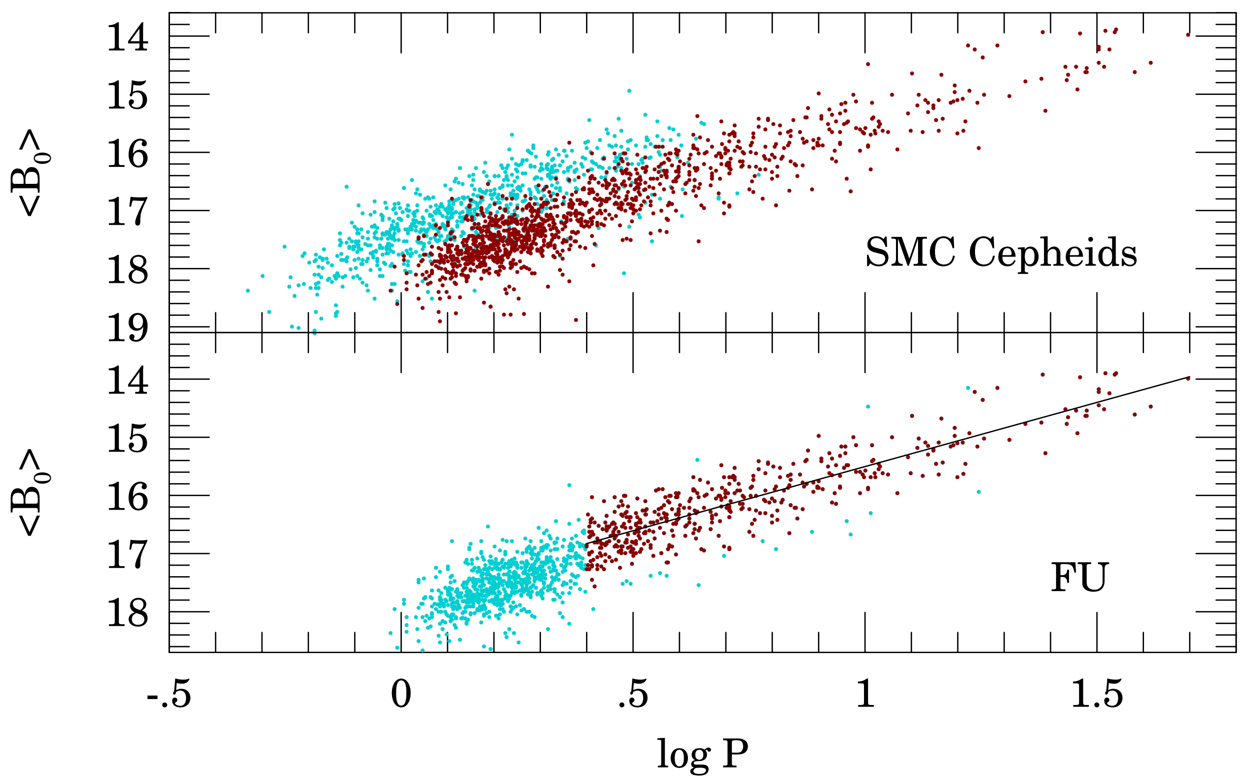

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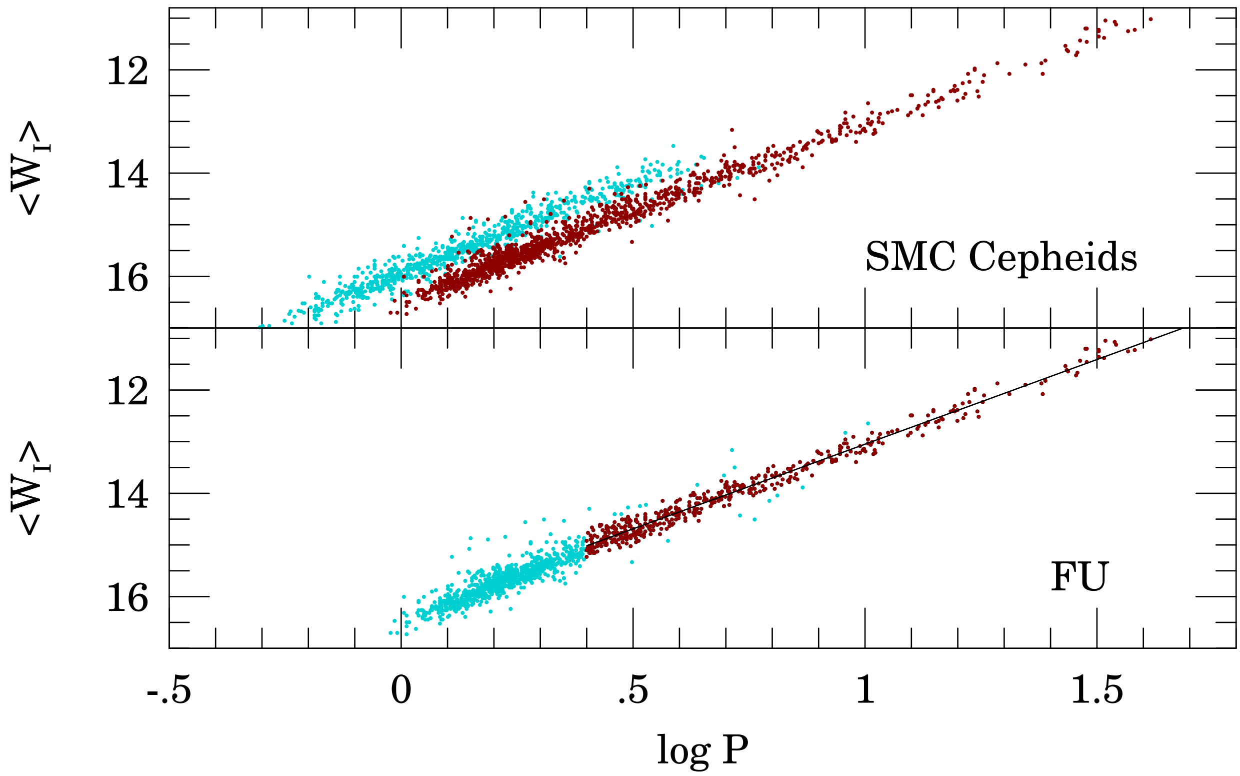

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